Extending SIESTA capabilities: removing field-periodic and stellarator symmetric limitations

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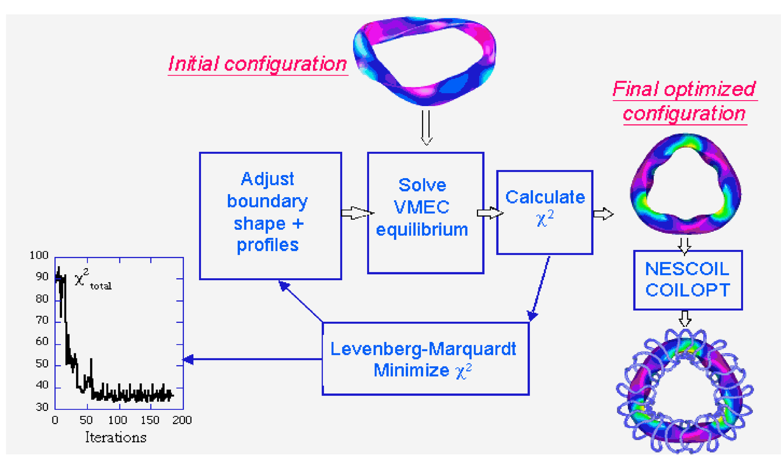
Overview

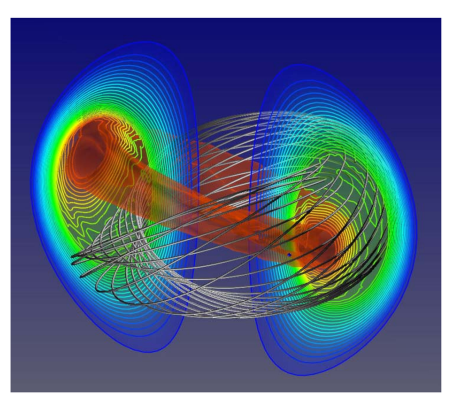
- SIESTA (Scalable Iterative Equilibrium Solver for Toroidal Applications) is a 3D nonlinear ideal MHD equilibrium solver capable of resolving islands in confinement devices in an accurate and scalable manner.
- The VMEC output is used as the initial equilibrium which is perturbed using the nonlinear energy minimization procedures of SIESTA.
- SIESTA has been used to demonstrate formation of islands in both tokamak and stellarator configurations.
- A new capability is being added which includes non-field-periodic perturbations, and thus can resolve lower-order magnetic islands due to error coils, etc. In order to do this, all of the toroidal modes must be included in the calculation.
- This will be tested on a CTH equilibrium that is unstable to low-order island formation.
- Further work will be done to allow SIESTA to calculate equilibria in configurations that are not stellarator symmetric. This will allow diverted tokamaks, such as DIII-D and ITER, to be analyzed properly.

Introduction to SIESTA

A fast (~mins), scalable 3D MHD equilibrium solver that resolves magnetic islands and stochastic regions would be useful for numerous applications

- STELLARATOR OPTIMIZATION.- include in optimization procedure at run-time (i.e., in STELLOPT suite).
- STELLARATOR/3D TOKAMAK EQUILIBRIUM RECONSTRUCTION.- (J. D. Hanson, V3FIT).
- EXTENDED-MHD SIMULATIONS.- initialize MHD simulations of neoclassical tearing modes at ITER-relevant resolutions.

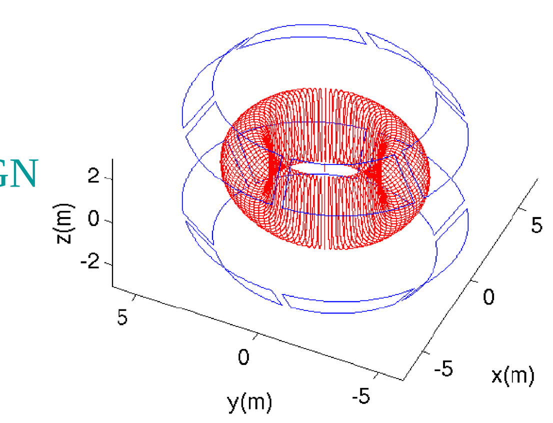




[Source: CEMM, PPPL]

Extended MHD simulation

- EXPERIMENT DESIGN and INTERPRETATION.- control of ELMs via resonant magnetic perturbations?



ELMs control on DIII-D

GUIDING PRINCIPLES.- avoid integration along magnetic field lines (slow/inaccurate) and retain scalability (to parallel environments). Also, build on previous ORNL expertise with 3D MHD codes (VMEC, COBRA…)

Basic equations of SIESTA

Ideal MHD energy (target function for minimization):

$$W=\int\left(\frac{B^2}{2\mu_0}+\frac{p}{\gamma-1}\right)dVol$$

The force in the unperturbed state which is not necessarily yet in equilibrium is given by:

$$\vec{F}_0=\vec{J}_0\times\vec{B}_0-\nabla p_0$$

For any plasma displacement, the linearized system becomes the following:

$$\delta\vec{F}=\delta\vec{J}\times\vec{B}_0+\vec{J}_0\times\delta\vec{B}-\nabla\delta p$$ Perturbed, linear system

$$\delta\vec{B}=\nabla\times\left(\vec{\xi}\times\vec{B}_0\right)$$ Faraday's law

$$\delta\vec{J}=\frac{1}{\mu_0}\left(\nabla\times\delta\vec{B}\right)$$ Ampere's law

$$\delta p=(\gamma-1)\vec{\xi}\cdot\nabla p_0-\gamma\nabla\cdot\left(p_0\vec{\xi}\right)$$ Mass conservation

To find equilibrium, minimize nonlinear force $\vec{F}\equiv\vec{J}\times\vec{B}-\nabla p=\vec{0}$ and update fields.

Finding the SIESTA equilibrium

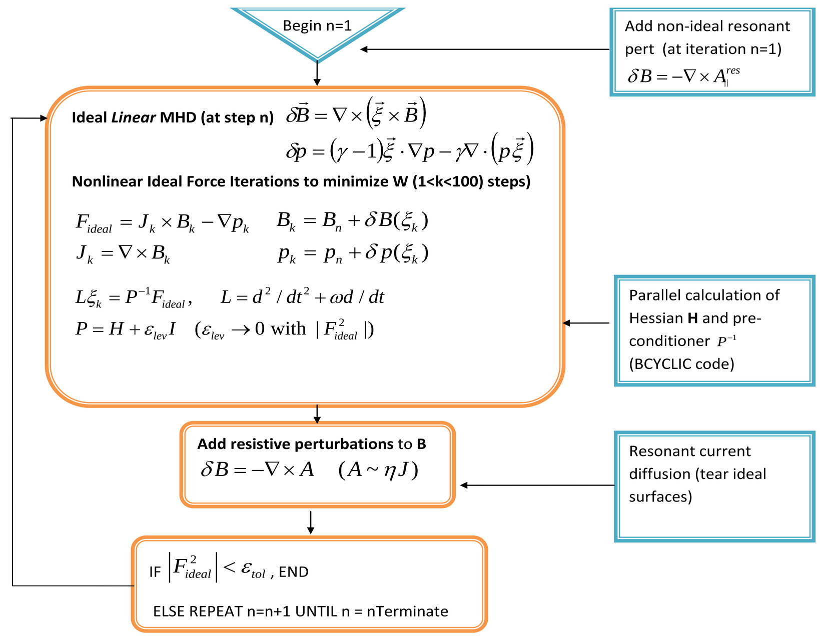
SIESTA finds the ideal MHD equilibrium in the presence of islands using Newton's method, a nonlinear minimization algorithm. Using this method, the energy is approximated locally as a quadratic function, obtained from the Taylor series expansion of the nonlinear energy. Minimizing the quadratic energy at each iteration is equivalent to solving the linearized MHD system and can be written as follows:

$$\vec{H}\vec{\xi}=-\vec{F}_{res} \qquad \frac{\partial\vec{F}}{\partial\vec{\xi}}\vec{\xi}=-\vec{F}_{res} \quad \text{where} \quad \frac{\partial W}{\partial\vec{\xi}}=-\vec{F}_{res}$$

Here the residual force is the nonlinear force evaluated at the previous displacement.

Beginning with a converged VMEC solution with closed, nested surfaces gives a starting equilibrium configuration "close" to the final equilibrium with islands. This potentially unstable equilibrium will contain current sheets at rational surfaces. Applying an initial resonant perturbation at the lowest-order rational surfaces may lead to island formation and a lower energy equilibrium. Finite resistivity is added to diffuse away the current sheets and allow the islands to form.

At each iteration of Newton's method, the linear MHD system must be solved for the new plasma displacement. Iterative solvers such as GMRES and variants of conjugate gradient work best for the inversion and solution of this linear system, due to the large condition number of the Hessian.



Work flow of a SIESTA simulation

Removal of field-periodicity requirement is underway

Current version of the code only includes toroidal mode numbers that are multiples of the number of field periods

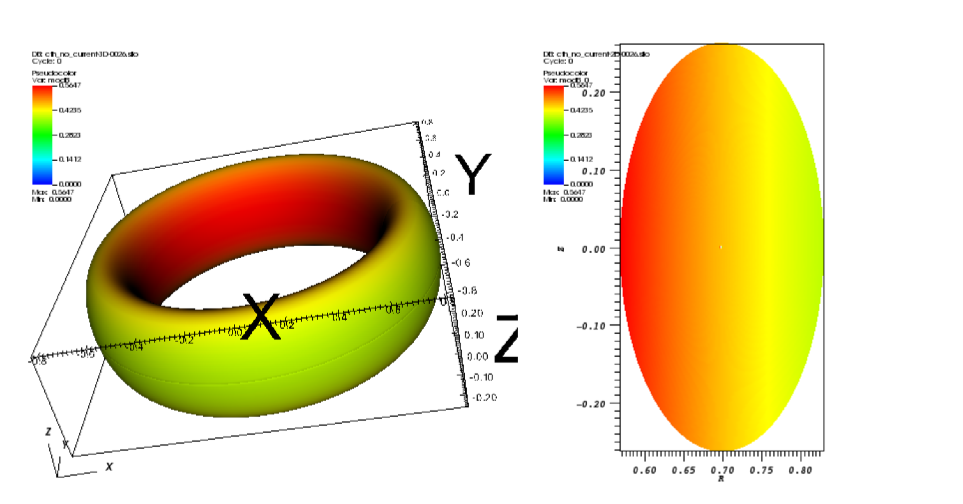
$$\xi^\rho(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}(\xi^\rho)_{mn}(\rho)\cos(m\theta+nN_{fp}\phi)$$
$$\xi^\alpha(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}(\xi^\alpha)_{mn}(\rho)\sin(m\theta+nN_{fp}\phi),\alpha\in\{\theta,\phi\}$$
$$B^\rho(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}(B^\rho)_{mn}(\rho)\sin(m\theta+nN_{fp}\phi)$$
$$B^\alpha(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}(B^\alpha)_{mn}(\rho)\sin(m\theta+nN_{fp}\phi),\alpha\in\{\theta,\phi\}$$
$$p(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}p_{mn}(\rho)\cos(m\theta+nN_{fp}\phi)$$

Only a single field period of the device needs to be modeled when this field-periodicity is enforced.

$$\xi^\rho(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}(\xi^\rho)_{mn}(\rho)\cos(m\theta+n\phi)$$
$$\xi^\alpha(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}(\xi^\alpha)_{mn}(\rho)\sin(m\theta+n\phi),\alpha\in\{\theta,\phi\}$$
$$B^\rho(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}(B^\rho)_{mn}(\rho)\sin(m\theta+n\phi)$$
$$B^\alpha(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}(B^\alpha)_{mn}(\rho)\sin(m\theta+n\phi),\alpha\in\{\theta,\phi\}$$
$$p(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}p_{mn}(\rho)\cos(m\theta+n\phi)$$

CTH (Compact Toroidal Hybrid) example equilibrium

CTH is a stellarator with current drive located at Auburn University. It will serve as a useful test case for this modification of the code because it is unstable to island formation with lower toroidal mode number than the number of field periods (5). This particular equilibrium has a 1/3 surface near the edge.



Mod|B| plot of CTH

Removal of stellarator symmetry requirement is planned

SIESTA uses a hybrid finite-differencing/Fourier representation of the fields in terms of the coordinates. Finite-differencing is used in the radial dimension while a Fourier expansion is done in both the poloidal and toroidal dimensions. Currently the code assumes stellarator symmetry for the fields used in SIESTA, resulting in the following parities for the Fourier series expansions of the quantities in terms of the angular coordinates.

$$\xi^\rho(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}(\xi^\rho)_{mn}(\rho)\cos(m\theta+n\phi)$$
$$\xi^\alpha(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}(\xi^\alpha)_{mn}(\rho)\sin(m\theta+n\phi),\alpha\in\{\theta,\phi\}$$
$$B^\rho(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}(B^\rho)_{mn}(\rho)\sin(m\theta+n\phi)$$
$$B^\alpha(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}(B^\alpha)_{mn}(\rho)\sin(m\theta+n\phi),\alpha\in\{\theta,\phi\}$$
$$p(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}p_{mn}(\rho)\cos(m\theta+n\phi)$$

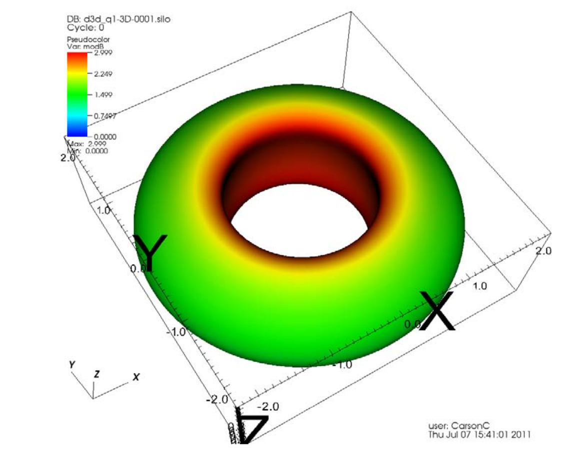
Removing the stellarator symmetric-only representation will allow for proper treatment of up-down asymmetric machines such as DIII-D.

Both parities will be required to model configurations that are not stellarator symmetric.

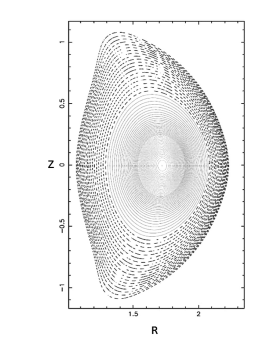
$$\xi^\beta(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}(\xi^\beta)^c_{mn}(\rho)\cos(m\theta+n\phi)+(\xi^\beta)^s_{mn}(\rho)\sin(m\theta+n\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}\underline{\xi}^\beta{}_{mn}(\rho)e^{i(m\theta+n\phi)},\beta\in\{\rho,\theta,\phi\}$$
$$B^\beta(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}(B^\beta)^c_{mn}(\rho)\cos(m\theta+n\phi)+(B^\beta)^s_{mn}(\rho)\sin(m\theta+n\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}\underline{B}^\beta{}_{mn}(\rho)e^{i(m\theta+n\phi)},\beta\in\{\rho,\theta,\phi\}$$
$$p(\rho,\theta,\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}p^c_{mn}(\rho)\cos(m\theta+n\phi)+p^s_{mn}(\rho)\sin(m\theta+n\phi)=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}\underline{p}_{mn}(\rho)e^{i(m\theta+n\phi)}$$
$$R=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}R_{mn}(\rho)\cos(m\theta+n\phi)\rightarrow R=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}R^c_{mn}(\rho)\cos(m\theta+n\phi)+R^s_{mn}(\rho)\sin(m\theta+n\phi)$$
$$Z=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}Z_{mn}(\rho)\sin(m\theta+n\phi)\rightarrow Z=\sum_{m=0}^{mpol}\sum_{n=-ntor}^{ntor}Z^c_{mn}(\rho)\cos(m\theta+n\phi)+Z^s_{mn}(\rho)\sin(m\theta+n\phi)$$

DIII-D (tokamak) example equilibrium

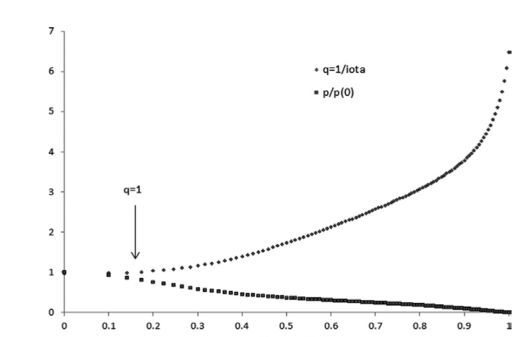
Removal of stellarator symmetry will allow us to accurately simulate DIII-D and other diverted tokamaks which do not have an "up-down" symmetry. Currently the code must use a symmetrized version of the configuration, as shown here.



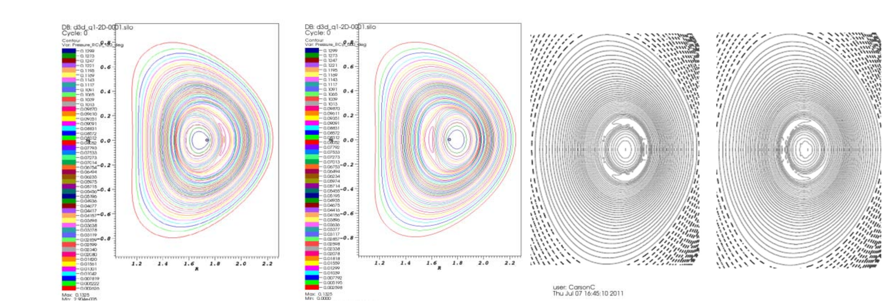
Magnetic field strength on plasma surface of DIII-D



Pressure contours of unperturbed VMEC DIII-D equilibrium



Safety factor profile demonstrating the q=1 surface at which the islands may form in SIESTA



Pressure contours of converged SIESTA equilibrium showing large q=1 island structure

Summary

- SIESTA has successfully been used to resolve islands in the equilibria of many configurations, including the DIII-D tokamak (shown here) and many stellarators.
- The field-period breaking version of the code is almost complete and will be tested on CTH .
- Future work will include the removal of the stellarator symmetry requirement currently implemented in the code. This will require a complex Fourier representation of field quantities and will allow for proper analysis of diverted tokamaks.

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