Calculations of Neoclassical Viscous Damping on Flux Surfaces Near Magnetic Islands in HSX

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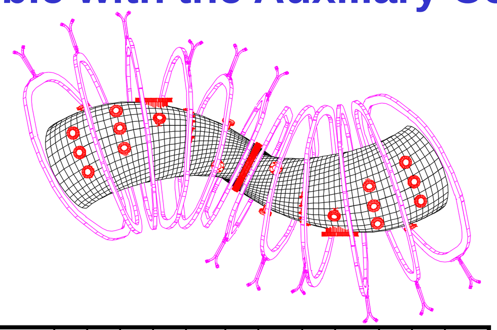
0. Contents:

- Configuration Flexibility in HSX
- Viscous Damping Near Magnetic Islands
- The Four Configurations of HSX under Study
- The Hamada Spectrum in these Configuration
- Enhanced Viscous Damping in the Vicinity of a Magnetic Island

1. Configuration Flexibility in HSX

Many Different Configurations are Accessible with the Auxiliary Coils



N4 Mirror (Standard)	+	+	+	-	-	-	-	-	-	+	+	+
N12 Mirror	+	-	-	+	+	-	-	+	+	-	-	+
Broad N Mirror	+	-	+	-	+	-	-	+	-	+	-	+
Hill	+	+	+	+	+	+	+	+	+	+	+	+

Percentage indicates the ratio of amp-turns in the two coil sets.

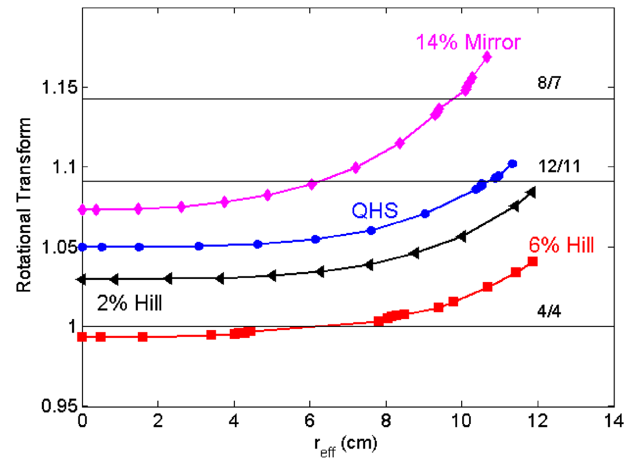
Configurations Discussed In This Poster:

QHS: Standard Quasisymmetric Configuration of HSX

Hill: Lowers Rotational Transform, Places Plasma on Magnetic Hill

Mirror: Large Toroidal Mirror (m=0, n≠0) Spectral Component is Present and Rotational Transform is Slightly Raised

Rotational Transform Profiles Indicate the Resonances



2. Viscous Damping Near Magnetic Islands

Magnetic Islands Change the Path of a Field Line

- Symmetry propertied are determined by the path of a magnetic field line through the |B| variation.
- Near a magnetic island, the magnetic surfaces are distorted.
- Hence, the field line path is changed near the magnetic island.
- This results in symmetry breaking on surfaces near the magnetic island.

Solve the Momentum Balance Equations on a Magnetic Surface

Poloidal and Parallel Momentum Balance

$$m_i N_i \frac{\partial}{\partial t} < \mathbf{B}_P \cdot \mathbf{U} > = -\frac{\sqrt{g} B^\zeta B^\alpha}{c} < \mathbf{J}_{plasma} \cdot \nabla\psi > - < \mathbf{B}_P \cdot \nabla \cdot \mathbf{\Pi} >$$

$$m_i N_i \frac{\partial}{\partial t} < \mathbf{B} \cdot \mathbf{U} > = - < \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi} >$$

Ampere's Law

$$\frac{\partial}{\partial t}\frac{\partial \Phi}{\partial \psi} < \nabla\psi \cdot \nabla\psi > = 4\pi\left(< \mathbf{J}_{plasma} \cdot \nabla\psi > + < \mathbf{J}_{ext} \cdot \nabla\psi > \right)$$

Represent the Magnetic Field Through The Hamada Spectrum

$$B = B_0 \sum_{n,m} b_{nm} \cos(m\alpha - n\zeta)$$

Use Linear Neoclassical Viscosities for the Plateau Regime

Plateau Viscosities From Shaing, Hirshman, and Callen, 1986

$$\langle B \cdot \nabla \cdot \Pi \rangle = \mu_\alpha U^\alpha + \mu_\zeta U^\zeta$$

$$\langle B_P \cdot \nabla \cdot \Pi \rangle = \mu_\alpha^{(P)} U^\alpha + \mu_\zeta^{(P)} U^\zeta$$

Viscosity Coefficients Are Calculated From Hamada Spectrum

$$\alpha_c = -\sum nm b_{n,m}^2 / |n - m\iota| \qquad \kappa = \pi^{1/2} P B_o / v_t B^\zeta$$

$$\alpha_P = \sum m^2 b_{n,m}^2 / |n - m\iota| \qquad \alpha_T = \sum n^2 b_{n,m}^2 / |n - m\iota|$$

$$\mu_\zeta = \kappa\left(B^\alpha \alpha_C + B^\zeta \alpha_T\right) \qquad \mu_\alpha = \kappa\left(B^\alpha \alpha_P + B^\zeta \alpha_\zeta\right)$$

$$\mu_\alpha^{(P)} = \kappa B^\alpha \alpha_P \qquad \mu_\zeta^{(P)} = \kappa B^\alpha \alpha_C$$

Modeling Gives Rise to Two Damping Rates

$$\left.\begin{matrix}\gamma_s \\ \gamma_f\end{matrix}\right\} = -\upsilon_{in} - \frac{\upsilon_1 - I_o \upsilon_{in}}{2\Omega} \pm \left[\left(\frac{\upsilon_1 - I_o \upsilon_{in}}{2\Omega} \right)^2 + \frac{\upsilon_{in} I_o \left(\iota \upsilon_\alpha + \upsilon_\zeta \right) + \upsilon_\zeta^{(P)} \upsilon_\alpha - \upsilon_\alpha^{(P)} \upsilon_\zeta}{\Omega} \right]^{1/2}$$

- The two rates correspond to the damping of flows in two directions on a magnetic surface
- The constants here are defined as:

$$\upsilon_1 = \upsilon_\alpha^{(P)} + (1 + I_o)\left(\iota \upsilon_\alpha + \upsilon_\zeta\right) - \frac{\langle \mathbf{B} \cdot \mathbf{B}_P \rangle}{\langle \mathbf{B} \cdot \mathbf{B} \rangle}\left(\upsilon_\alpha^{(P)} + q \upsilon_\zeta^{(P)}\right) - \frac{\langle \mathbf{B} \cdot \mathbf{B}_P \rangle}{\langle \mathbf{B}_P \cdot \mathbf{B}_P \rangle} \iota \upsilon_\alpha$$

$$I_o = \left(B^\alpha / 2\pi\right)^2 \langle \nabla\rho \cdot \nabla\rho \rangle / \left(4\pi m_i N_i \langle \mathbf{B}_p \cdot \mathbf{B}_p \rangle\right)$$

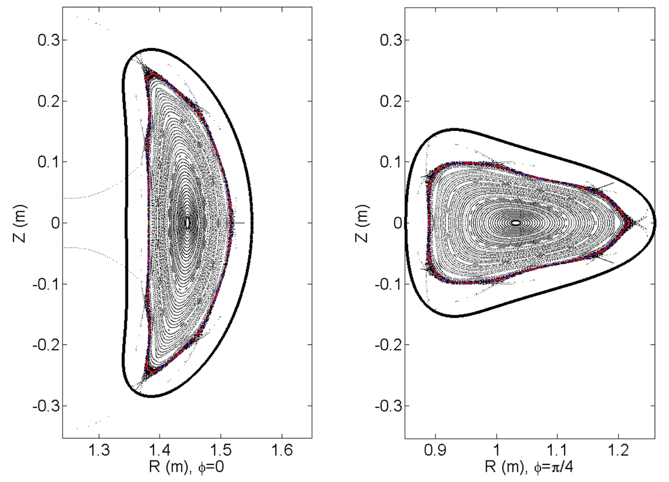
$$\Omega = 1 + I_o - \langle \mathbf{B} \cdot \mathbf{B}_p \rangle^2 / \left(\langle \mathbf{B}_p \cdot \mathbf{B}_p \rangle \langle \mathbf{B} \cdot \mathbf{B} \rangle\right)$$

$$\upsilon_\zeta^{(P)} = \mu_\zeta^{(P)} B^\alpha / m_i N_i \langle \mathbf{B}_p \cdot \mathbf{B}_p \rangle \qquad \upsilon_\alpha^{(P)} = \mu_\alpha^{(P)} B^\alpha / m_i N_i \langle \mathbf{B}_p \cdot \mathbf{B}_p \rangle$$

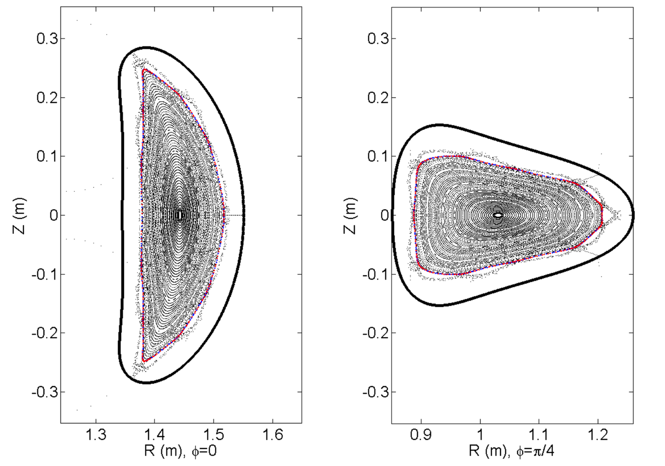
$$\upsilon_\zeta = \mu_\zeta B^\zeta / m_i N_i \langle \mathbf{B} \cdot \mathbf{B} \rangle \qquad \upsilon_\alpha = \mu_\alpha B^\zeta / m_i N_i \langle \mathbf{B} \cdot \mathbf{B} \rangle$$

3. Four Configurations of HSX

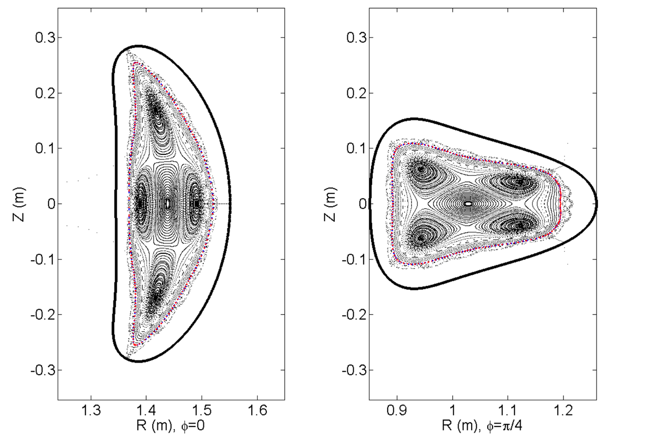
QHS: Standard Configuration with 12/11 Resonance



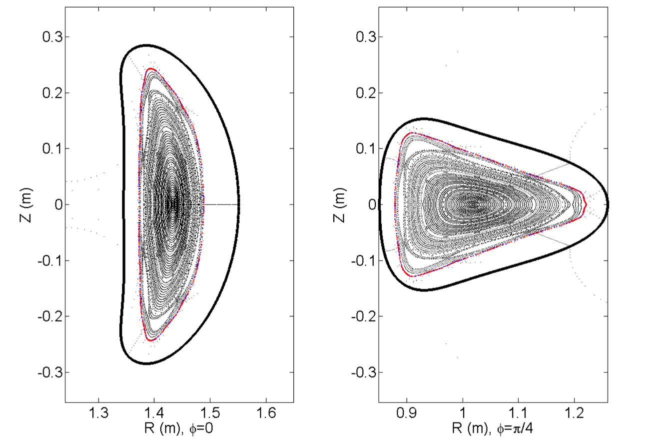
2% Hill, Transform Lowered To Avoid 12/11 Resonance



6% Hill, Transform Lowered to include 4/4 Resonance

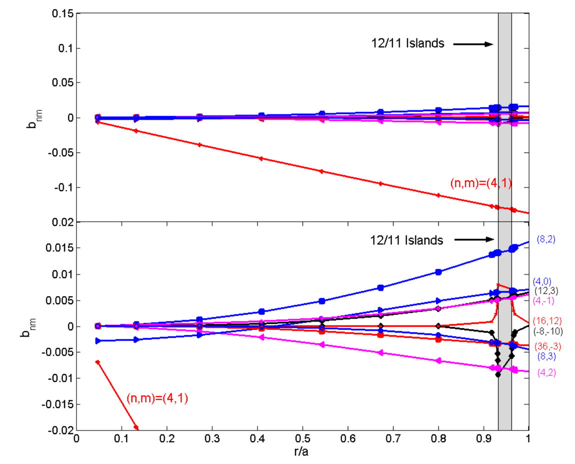


14% Mirror: Large Toroidal Ripple and 8/7 Resonance



4. The Hamada Spectrum is Modified Near an Island

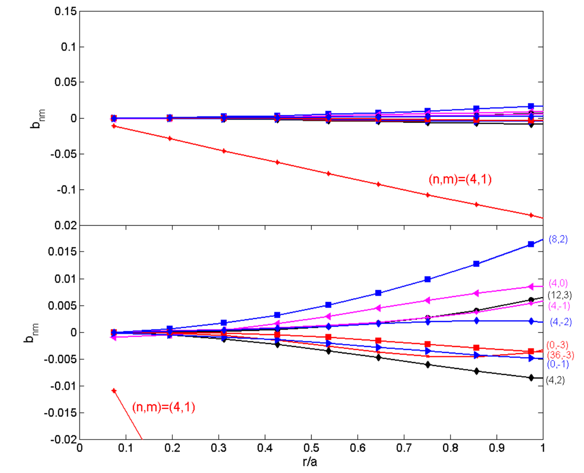
QHS: Small Symmetry Breaking in Vicinity of 12/11



10 Largest Spectral Components at r/a=0.93. Islands Exist: 0.931<r/a<0.961

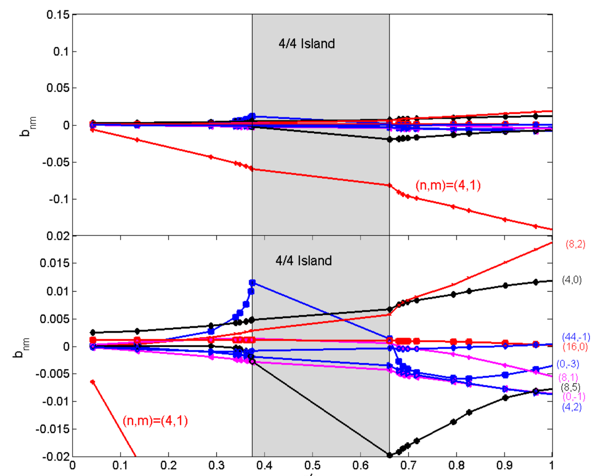
$$(4,1) \pm (12,11) = \begin{cases} (16,12) \\ (-8,-10) \end{cases}$$

2% Hill, No Low Order Resonaces



10 Largest Spectral Components at r/a=0.8 No Resonances with n<16, where $\iota/2\pi = n/m$

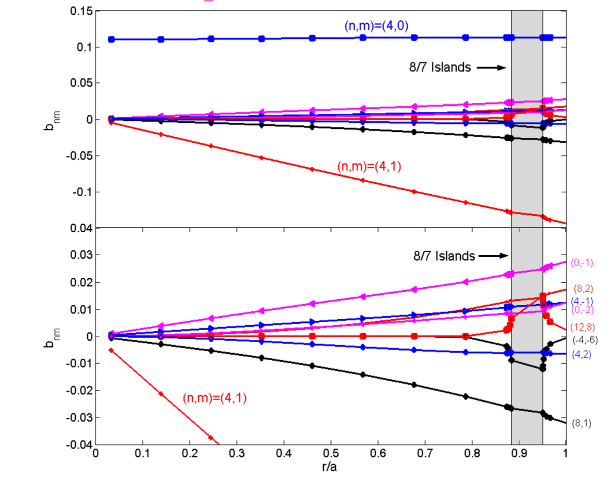
6% Hill, Large Symmetry Breaking near 4/4 Resonance



10 Largest Spectral Components at r/a=0.37 Islands Exist: 0.375<r/a<0.661

$$(4,1) \pm (4,4) = \begin{cases} (8,5) \\ (0,-3) \end{cases}$$

14% Mirror: Toroidal Mirror Swamps Island Induced Effects



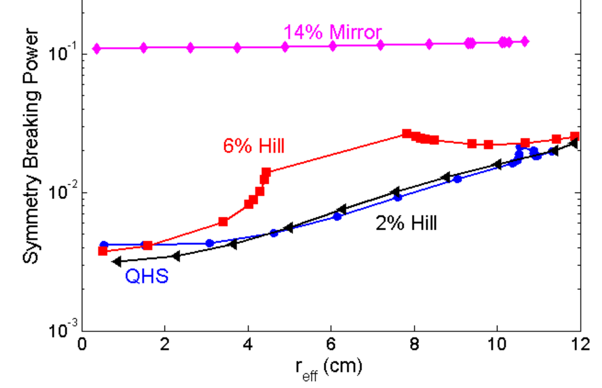
10 Largest Spectral Components at r/a=0.88 Islands Exist: 0.883<r/a<0.949

$$(4,1) \pm (8,7) = \begin{cases} (12,8) \\ (-4,-6) \end{cases}$$

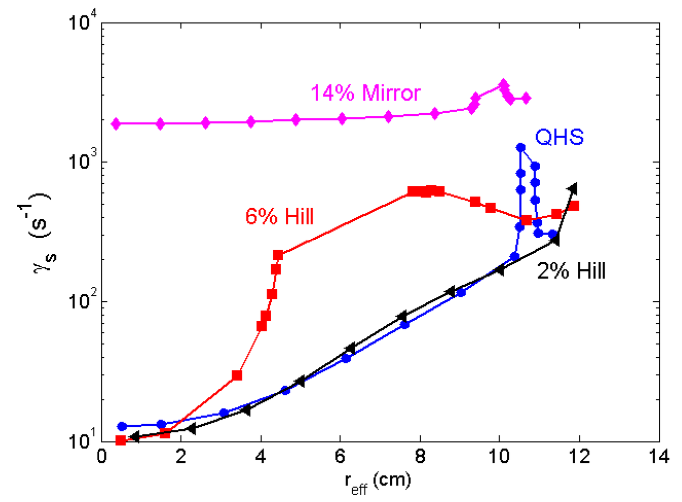
5. The Viscous Damping is Increased Near the Island

Symmetry Breaking Increases Near the Magnetic Islands

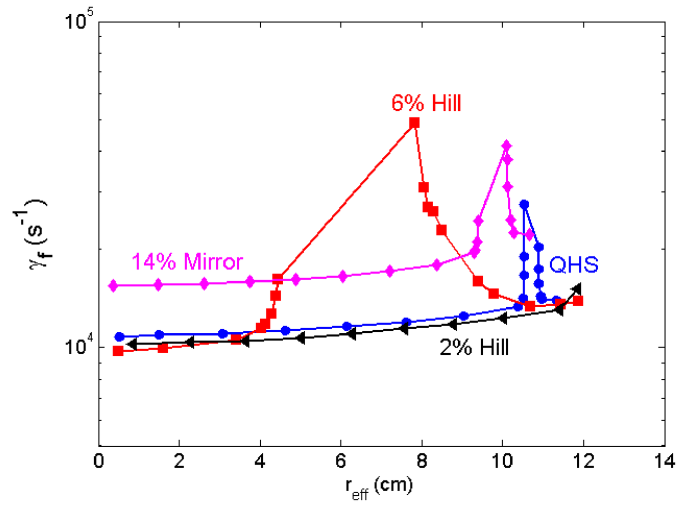
$$\text{Power} = \left(\sum_{\substack{(n,m) \neq (0,0) \\ n/4 \neq m}} b_{n,m}^2 \right)^{1/2}$$



Slower Damping Rare is Changed in Vicinity of Magnetic Island



Faster Damping Rare is Strongly Affected in Vicinity of Magnetic Island



This is expected to be the strongest experimental signature.

These results accepted for publication in Physics of Plasmas