3D Reconstruction and Modeling of Equilibrium Currents in the HSX Stellarator

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Overview

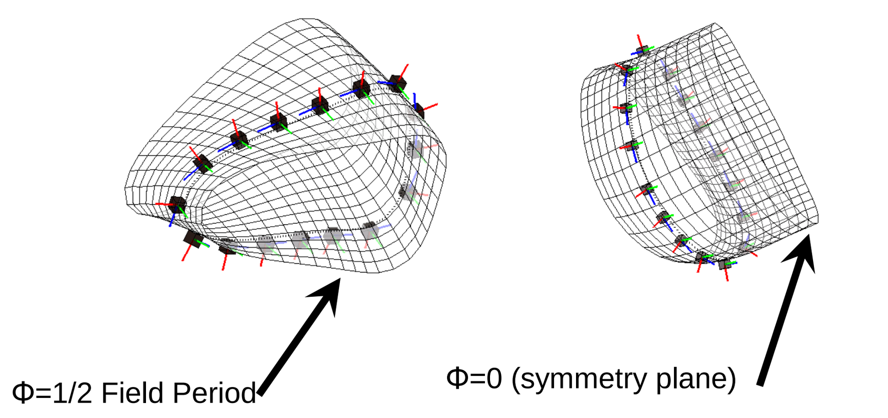
The Helically Symmetric eXperiment

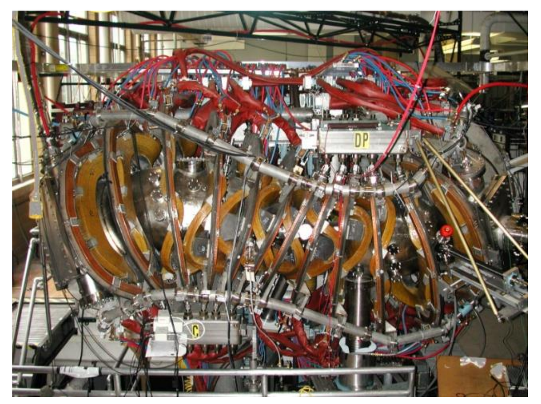
- is Quasi-Helically Symmetric and has almost no toroidal curvature
- B_0 = 1 Tesla
- 26 kW – 100 kW ECR Heating (1st Harmonic O-mode)
  - Perpendicular launch
- Toroidal current is predominantly bootstrap-driven
  - Flows in opposite direction and reduced compared to a tokamak
  - Reverses with B-field direction
  - Induces toroidal current with long decay times: $\tau_{\eta/\mu_0} \geq \tau_{EXP}$
- Predicting/controlling the toroidal current profile is important for the next generation of stellarators.
- 3D equilibrium reconstruction is important for stellarators, RFPs and tokamaks with non-axisymmetric fields.
- The steady-state neoclassical bootstrap current is calculated by PENTA[1] and the evolving current profile is modeled by a diffusion equation with a 3D susceptance matrix[2] (forward model). The magnetic signals for the model plasma profile are computed by V3FIT[3].
- 3D reconstruction of plasma and current profiles are performed with V3FIT.

Special thanks to Jim Hanson, Steve Knowlton and the entire V3FIT team!!!

Diagnostics

- 2 poloidal arrays of 16 triplets measure local changes in the magnetic field
  - Mounted on the exterior of the vacuum chamber
  - Recently modified to reduce noise in measured signal
- Rogowski coils (internal and external) measure the net toroidal current
- Toroidal Flux loops: Sensitive mainly to pressure-driven diamagnetic flux
- Internal poloidal and toroidal Mirnov arrays
- Thomson Scattering, Microwave Interferometer, CHERS
- A carbon graphite limiter is placed at the vacuum LCFS





A single "triplet" diagnostic

¾" cube

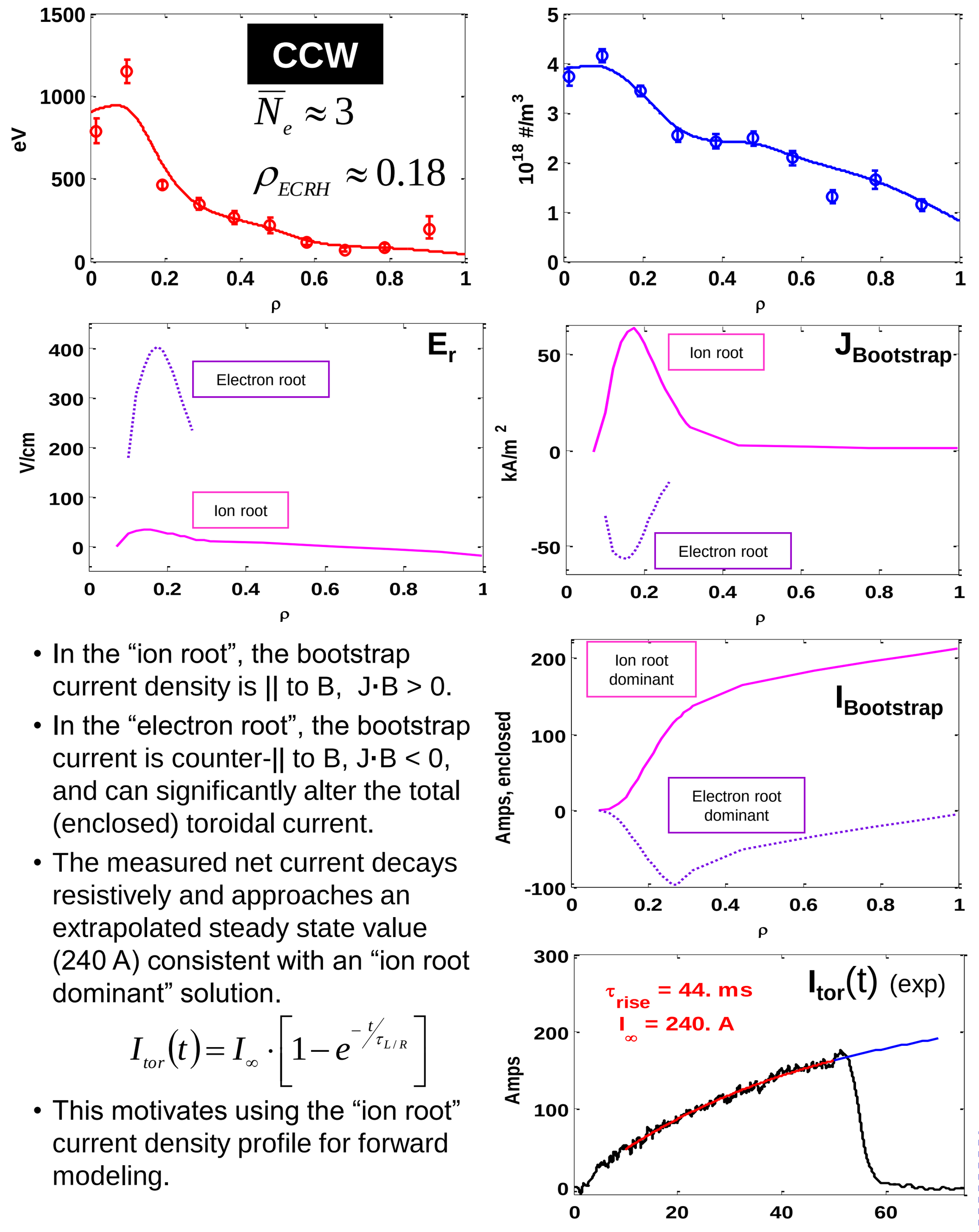
Photo courtesy P. Probert

References

[1] D.A. Spong, Phys. Plasmas 12, 056114 (2005).
[2] P. I. Strand and W. A. Houlberg, Phys. Plasmas 8, 2782 (2001).
[3] J.D. Hanson, et al, Nucl. Fusion 49 (2009) 075031.

Forward Modeling

PENTA Uses measured plasma profiles (Te, Ne, Ti, Ni) and predicts the ambipolar radial electric field and steady-state parallel flows and bootstrap current. Includes the effects of momentum conservation between plasma species.



- In the "ion root", the bootstrap current density is || to B, J·B > 0.
- In the "electron root", the bootstrap current is counter-|| to B, J·B < 0, and can significantly alter the total (enclosed) toroidal current.
- The measured net current decays resistively and approaches an extrapolated steady state value (240 A) consistent with an "ion root dominant" solution.

$$I_{tor}(t) = I_\infty \cdot \left[1 - e^{-t/\tau_{L/R}}\right]$$

- This motivates using the "ion root" current density profile for forward modeling.

Time Evolution

3d susceptance matrix links toroidal and poloidal currents and magnetic fluxes

$$\mu_0 \begin{pmatrix} I \\ F \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} \Psi' \\ \Phi' \end{pmatrix}$$

$S_{12} = S_{21} = 0$ for Tokamaks

$S_{11} \approx S_{12} \approx S_{21}$ for HSX

1-D diffusion equation for rotational transform

$s = \Phi/\Phi_{LCFS}$ ′ $= d/ds$

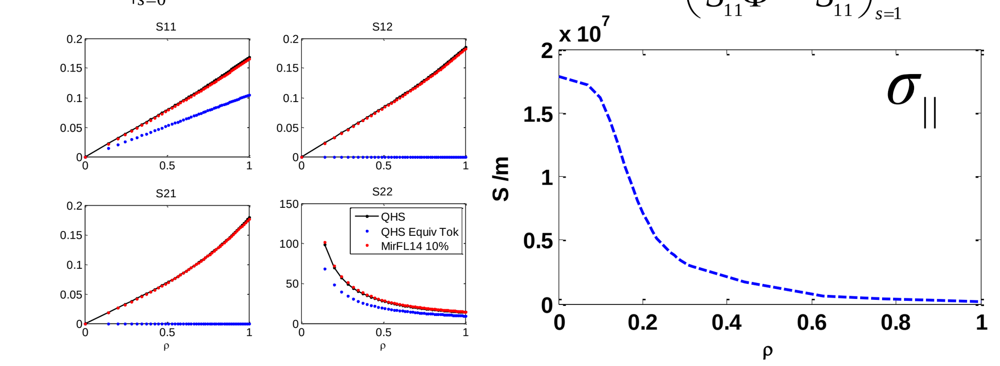
Any non-inductive source

$$\frac{d\iota}{dt} = \frac{1}{\Phi_{LCFS}^2}\frac{d}{ds}\left(\eta V'\left[\frac{\langle B^2\rangle}{\mu_0}\frac{d}{ds}(S_{11}\iota + S_{12}) + p'(S_{11}\iota + S_{12}) - \langle \mathbf{J}\cdot\mathbf{B}\rangle\right]\right)$$

Boundary conditions

On axis: finite current density $\left.\frac{d\iota}{ds}\right|_{s=0} = 0$

@ LCFS : set by measurement $\left.\iota\right|_{s=1} = \left(\frac{\mu_0 I}{S_{11}\Phi'} - \frac{S_{12}}{S_{11}}\right)_{s=1}$



The evolved current and measured profiles serve as an 'initial guess' for the reconstruction

Equilibrium Reconstruction

V3FIT uses VMEC for the equilibrium calculation. The pressure and current profiles each have four parameters which can be adjusted during the reconstruction step. The net toroidal flux is another fit parameter (total fit parameters = 9).

Pressure profile

$$p(s) = prescale \cdot \left[\frac{1}{N_0}\left(\left(1+\left(\frac{s}{a_2^2}\right)^{a_3}\right)^{-a_4} - c_0\right)\right]$$

$$N_0 = 1 - c_0 \text{ and } c_0 = \left(1+\left(\frac{1}{a_2^2}\right)^{a_3}\right)^{-a_4}$$

Enclosed current profile

$$I_{enclosed}(s) = curtor \frac{2}{\pi}\arctan\left(\frac{a_1 s^{a_2}}{(1-s)^{a_4}}\right)$$

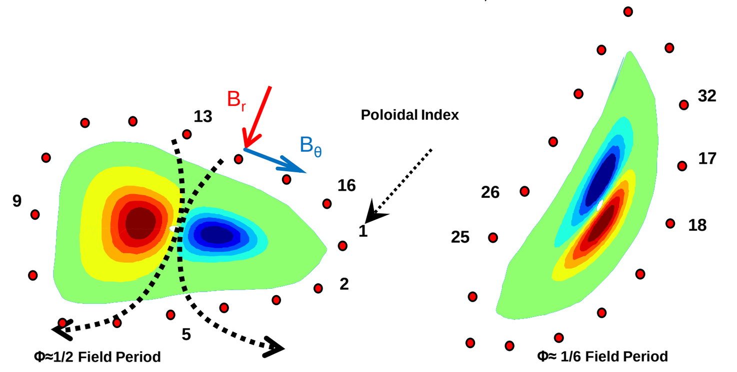
- V3FIT adjusts the profile parameters to minimize the difference between the measured and modeled magnetic signals and limiter position.

$$g^2(p) = \sum_i \kappa_i \left(\frac{S_{o,i} - S_{m,i}(p)}{\sigma_i}\right)^2 \qquad g_\nu^2 = g^2/\nu$$

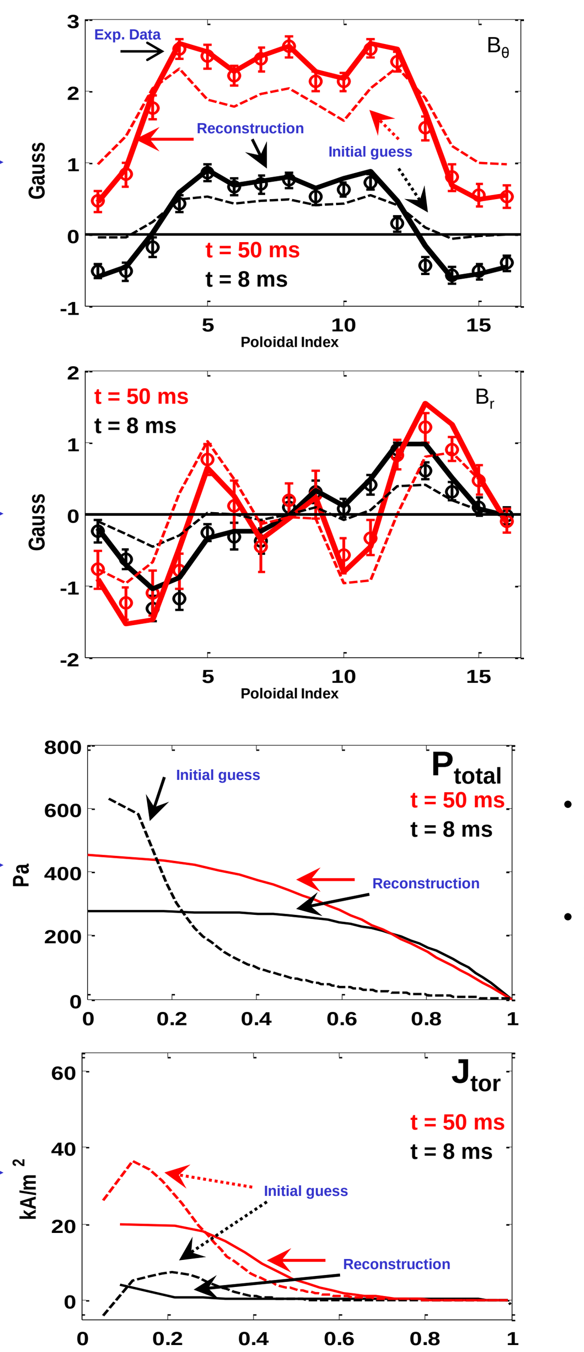
ν : # of degrees of freedom, $\nu = N - m - 1$, N = # of observations, m = # of fit parameters

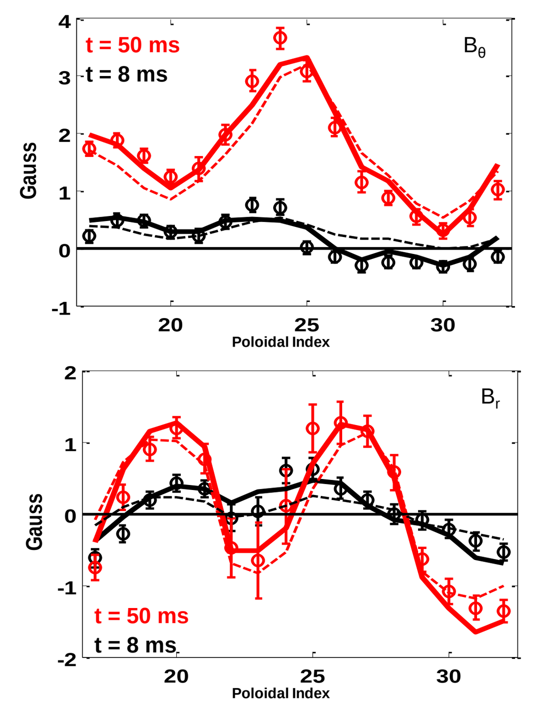
g_ν^2 is analogous to the "goodness of fit" reduced chi-squared

$g_\nu^2 \approx 1$: Fit function is a good approximation $\quad g_\nu^2 >> 1$: Indicates a poor model fit



Measured, initial guess and reconstructed signals

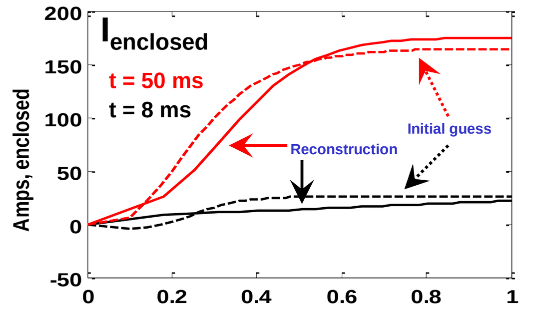




t = 50 ms $g^2_{v,initialguess} = 12.3$ $g^2_{v,reconstruction} = 6.8$

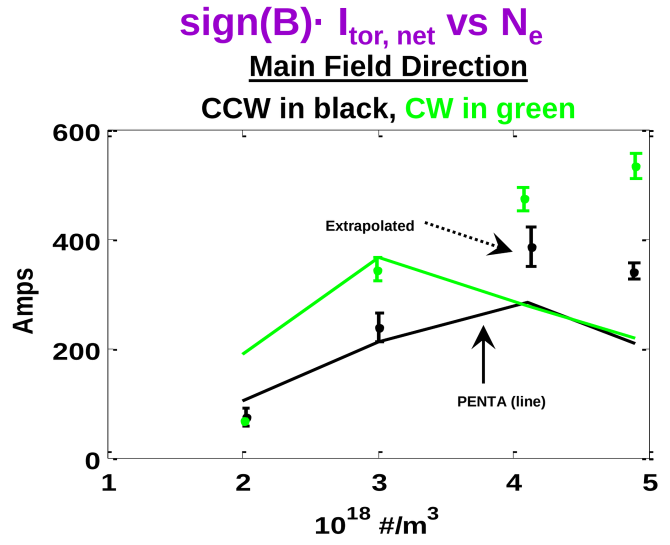
t = 8 ms $g^2_{v,initialguess} = 16.6$ $g^2_{v,reconstruction} = 10.4$

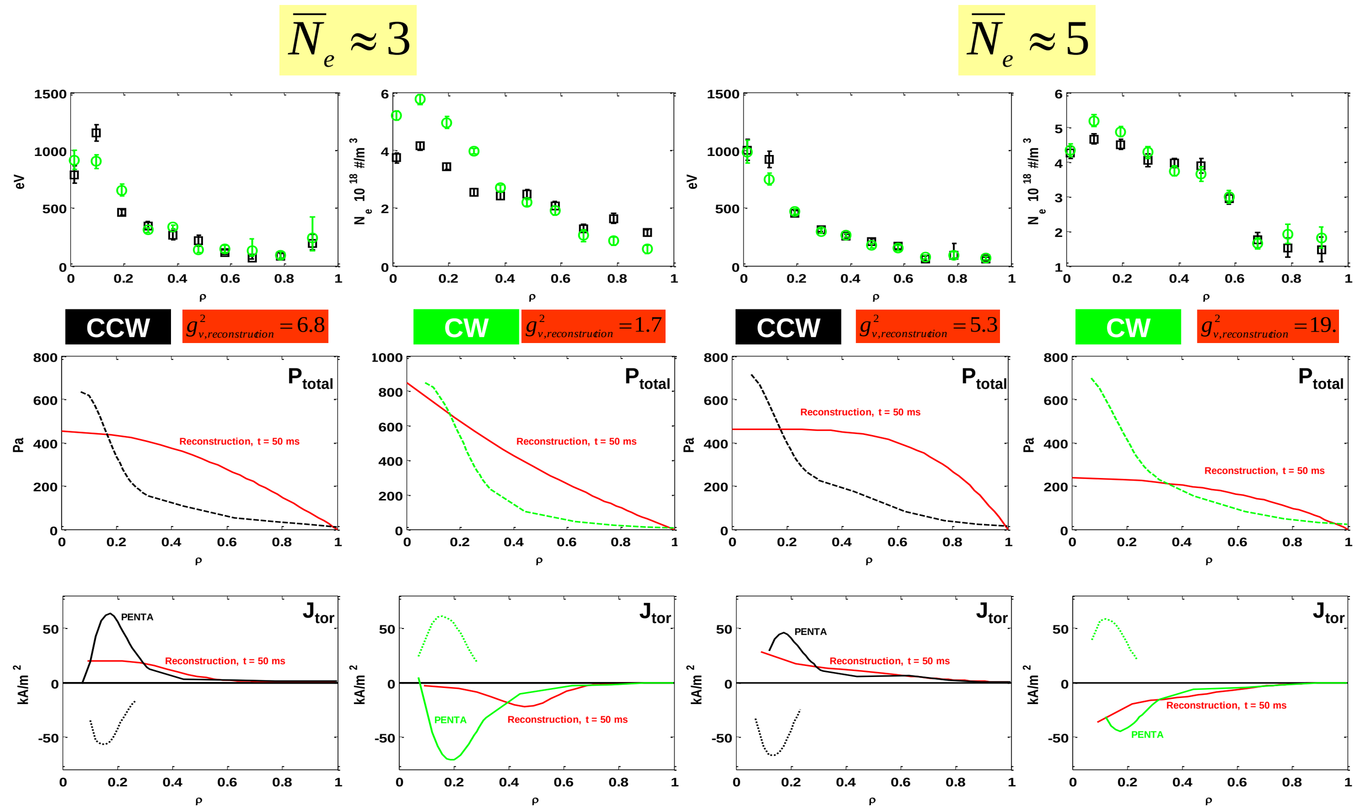
- Early in time, the dipole-field from the Pfirsch-Schlüter current is seen. Later in time, the effects of the bootstrap current dominate.
- The reconstructed pressure profiles evolve and are broader than those derived from Thomson Scattering, which are stationary after initial plasma breakdown.



Flip Field experiments reveal non-symmetric component of net current

- With the ECRH location fixed at $\rho_{ECRH} \approx$ +0.18, the line-averaged line density was varied from 2 – 5 x 10[18] #/m[3].
- At low density, the net current is similar for each direction, but as the density is raised, more current is measured with the field in the CW direction. The difference increases with density.
- For $N_e \geq 4 \times 10^{18}$ #/m^3, the extrapolated total (net) current exceeds the neoclassical estimate by up to x2





With N_e ~ 3 …

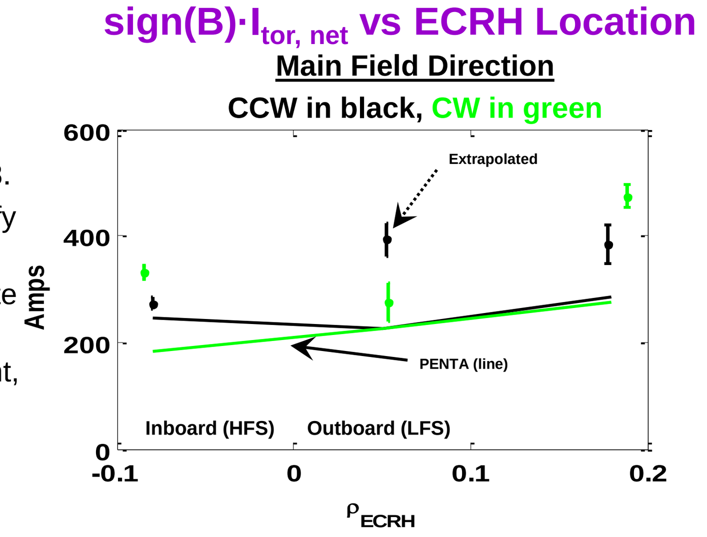
- and **B in the CCW direction**, the parallel current is approaching the neoclassical ion-root estimate.
- and **B in the CW direction**, the parallel current on-axis is reduced compared to the neoclassical ion-root estimate.

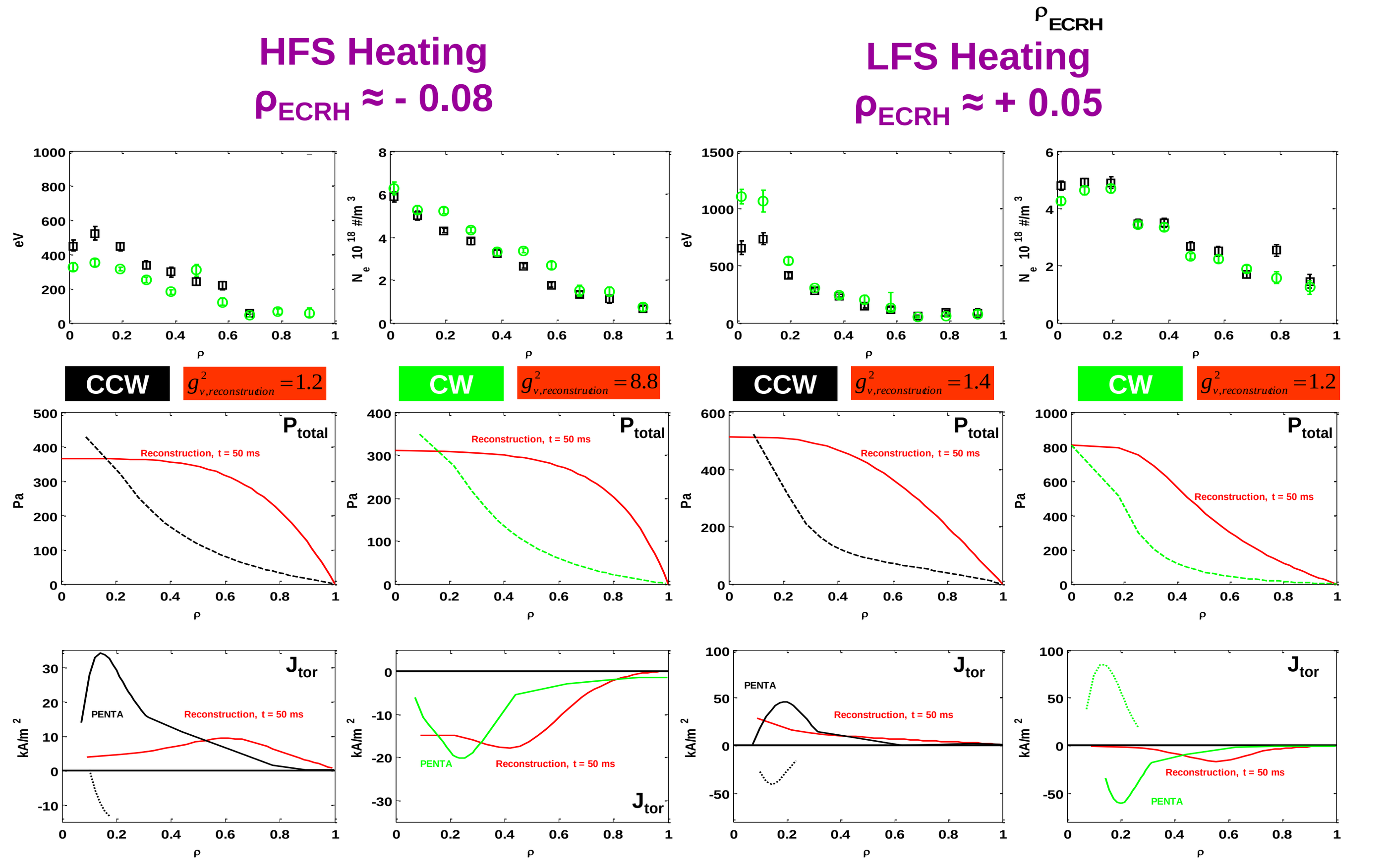
With N_e ~ 5 …

- The parallel current density is similar to the neoclassical ion-root estimate regardless of field direction.

Near-axis ECRH alters the local current density

- The ECRH resonance location was scanned from inboard (HFS) heating to outboard (LFS) heating.
- Maximum central T_e is achieved with $\rho_{ECRH} \approx$ +0.18.
- The direction of the main field is reversed to identify any 'non-symmetric' contribution or effects
- The neoclassical calculation tends to underestimate the net toroidal current (extrapolated).
- To compensate for the lack of non-inductive current, the Forward Model requires a toroidal electric field to drive the excess current (not shown).





With HFS heating …

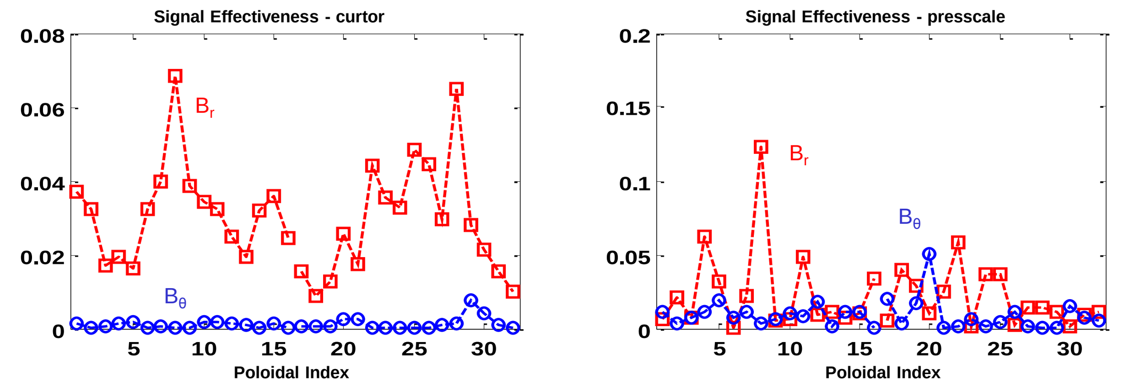
- and **B in the CCW direction**, the parallel current is reduced on-axis, compared to the neoclassical estimate.
- and **B in the CW direction**, the parallel current on-axis is similar to the neoclassical estimate.

With LFS heating …

- and **B in the CCW direction**, the parallel current on-axis is similar to the neoclassical estimate.
- and **B in the CW direction**, the parallel current is reduced on-axis compared to the neoclassical estimate.

- The 'Signal Effectiveness' is a measure of the impact that a diagnostic has on a particular reconstruction parameter

$$R_{ji} = \frac{d\ln\sigma_{pj}}{d\ln\sigma_i} = \frac{\sigma_i}{\sigma_{pj}}\frac{d\sigma_{pj}}{d\sigma_i}$$



Future Work

- Refine reconstruction
  - Adjust fit parameterization to improve the 'goodness-of-fit' $I_{enclosed}(s) = a_0 + \sum_{i=1}^{2} a_{1+4(i-1)}\frac{2}{\pi}\arctan\left(\frac{a_{2+4(i-1)}s^{a_{3+4(i-1)}}}{(1-s)^{a_{4+4(i-1)}}}\right)$
  - Increase radial grid density/resolution near axis
- Identify the source of the asymmetric current density
  - Ion-root vs. electron-root effects? ECH-driven?
- Automate reconstruction process
- Incorporate more diagnostics in reconstruction
  - Thomson Scattering, Soft X-ray, etc.
- Improve modeling of induced fields/eddy currents. Improve diagnostics, if necessary.
- Impose vacuum magnetic islands and study plasma equilibrium and/or transport response.
  - See **C.R Cook UP9.00088** "Application of SIESTA to Well and Hill Equilibria in HSX"

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